

INNOVATIVE ROOF SYSTEM FOR LOW-COST HOUSING

K Born^{*}, G Hocking[†] and W Mansoor[‡]

Industry Representatives

A Fitchett[§] and R Bradley[¶]

Study Group Participants

M Mkhize, Z Xulu, T Duba, G Marewo, D P Mason,
G Fareo, G Hocking, K Born, M S Alhaj and C M Khalique

Abstract

Large numbers of South Africa's poor still live in informal housing. The housing is often makeshift and almost always poorly suited to the conditions. A prototype for low-cost housing was designed and built on the campus of the University of Witwatersrand and monitored for several years with regard to its internal temperature profile. The Study Group was asked to model the performance with a view to improving or optimising the design. Two models, a simple one-dimensional model and a finite-element model were constructed and implemented on the design. The one-dimensional model incorporated real weather data and a simplified internal structure. A novel shape factor was used to consider different shapes for the roofing. In the finite-element model, the internal temperatures were modeled by assigning the external roof temperature and adjusting until the match with internal temperatures was good. Both models had some success in reproducing the internal thermal behavior of the housing. A recommendation for future work is that the two models be combined into a full simulation of the thermal properties of the structure.

1 Introduction

Large numbers of South Africa's poor still live in informal housing. The housing is often makeshift and almost always poorly suited to the conditions. Our industry partners built a

^{*}School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa. *email:* kendall.born@wits.ac.za

[†]Department of Mathematics and Statistics, Murdoch University, Perth, Western Australia, Australia. *email:* g.hocking@murdoch.edu.au

[‡]Department of Mathematics, College of Education for Pure Science Ibn-Al Haitham, University of Baghdad Iraq.

[§]School of Civil and Environmental Engineering, University of the Witwatersrand, Johannesburg. *email:* anne.fitchett@wits.ac.za

[¶]School of Civil and Environmental Engineering, University of the Witwatersrand, Johannesburg. *email:* ryan.bradley@wits.ac.za

prototype low-cost house on the West Campus of the University of the Witwatersrand, using cement stabilized earth for the curved (double-vaulted) roofs and recycled building materials for the brick walls. This structure was built not only to test a possible form of cheap housing, but also to provide a more comfortable thermal environment for the occupants. This prototype

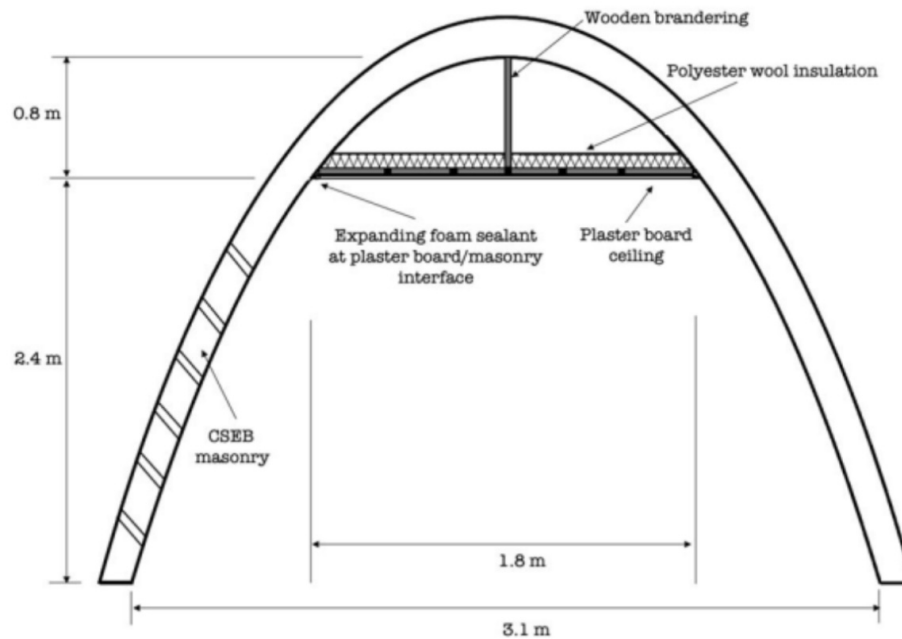


Figure 1: The prototype house on the West Campus of the University of Witwatersrand used in the experiments (top) and the roof design (bottom).

was monitored over a number of years, with data collected for temperatures in the northern and southern rooms and outside, including scenarios of open and closed windows and different coloured roofing. The group was charged with doing some modeling to determine the effectiveness of this design and to estimate heat exchange through the roof and throughout the building, with a view to doing more complex modeling of air flow in the house, potentially identifying design improvements. Several models for the thermal performance of the building and roof structure were considered, but the group finally settled on two different approaches. The first was to write a simple, vertical one-dimensional model that incorporated heating from the sun,

radiative transfer, and heat exchange through the roof, and the second was a finite element model that included a shaped roof and both sides of the two-domed structure. Both models were used to make preliminary calculations of the heat in the structure for comparison with the data.

The one-dimensional model included a shape-factor function to take into account the different possible shapes of the roof structure, from flat to domed, and resulting computations of internal temperatures were qualitatively consistent with the measurements. However, this model was limited to a single domed structure. The finite element model performed well in reproducing the heating phases of the situation for the double roof structure, but initially heat storage in the roof was not included, and so night time temperatures were sometimes underestimated. In this model, the temperature on the outside of the roof was imposed and fitted rather than computed from the ambient weather conditions.

2 Overview of the data

Data was collected at the site throughout the year as the house was set up in different ways. A glimpse of the data can be seen in Figures 2-6, and shows the heat difference between indoors and outdoors over a single day. This includes situations such as when the windows are open, when the house is occupied, and when the season changes.

Data in the plots shows temperatures measured in the house over 24 hours with the house in different configurations of colour and venting, and at different times of the year. Some observations include;

- The roof colour makes a difference to heat absorption, e.g. a white roof reflects a lot of heat, leaving cooler rooms,
- Keeping the windows closed in summer seems to provide a more stable environment, but may trap heat in at night,
- The delay between interior and exterior temperature signals is around 2 hours during the heating phase, but longer during the cooling,
- Performance in summer is beneficial to comfort, but in winter the greater insulation may lead to a colder environment. However, it is still more comfortable than the informal housing which would be closer to the outside temperature in the absence of heating. In winter, the south side is consistently colder than the north due to it being sheltered from direct sunlight by the northern dome.
- Temperature on the ground floor is very stable throughout each period of the year. During summer it is fairly stable in the mid-twenties even when outside temperatures go very high. Similarly, in winter it is stable at around 15°C even when outside temperatures are quite low.

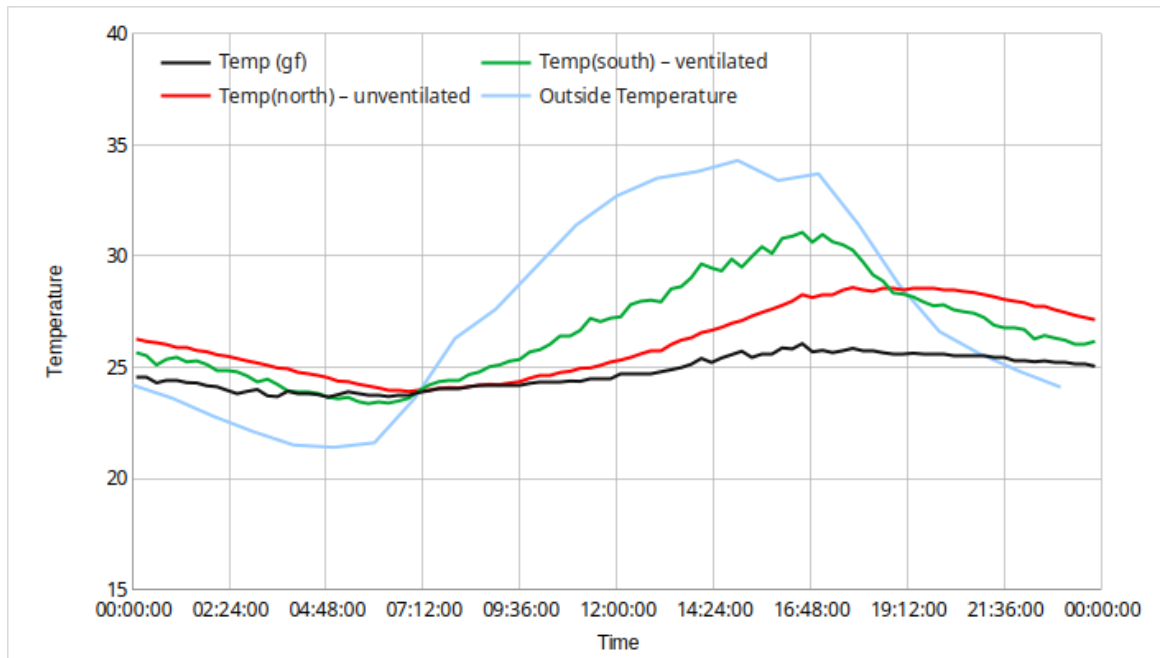


Figure 2: Temperature ($^{\circ}C$) data over 24 hours during summer (26/12/2018) with white roof. North unventilated, south ventilated. The south side is warmest, possibly due to open windows allowing in the heat. Ground floor (gf) is relatively stable around $25^{\circ}C$.

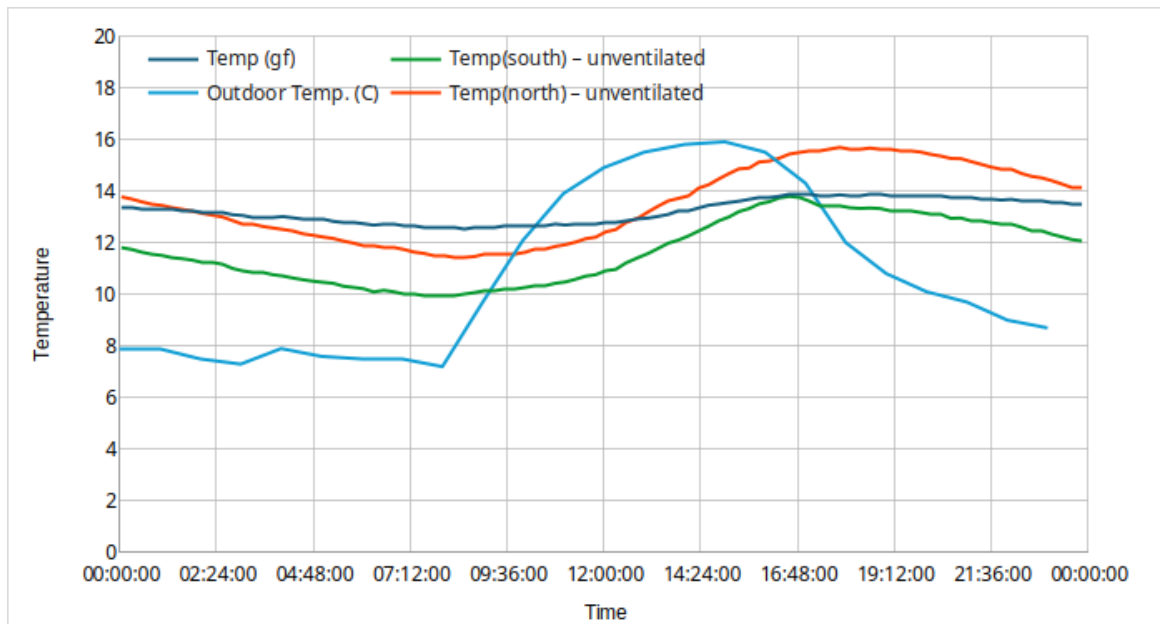


Figure 3: Temperature ($^{\circ}C$) over 24 hours during winter (11/06/2019) with white roof and no ventilation. The north side is consistently warmer due to the sheltering of the south by the northern dome. The Ground floor (gf) is relatively stable around $13^{\circ}C$.

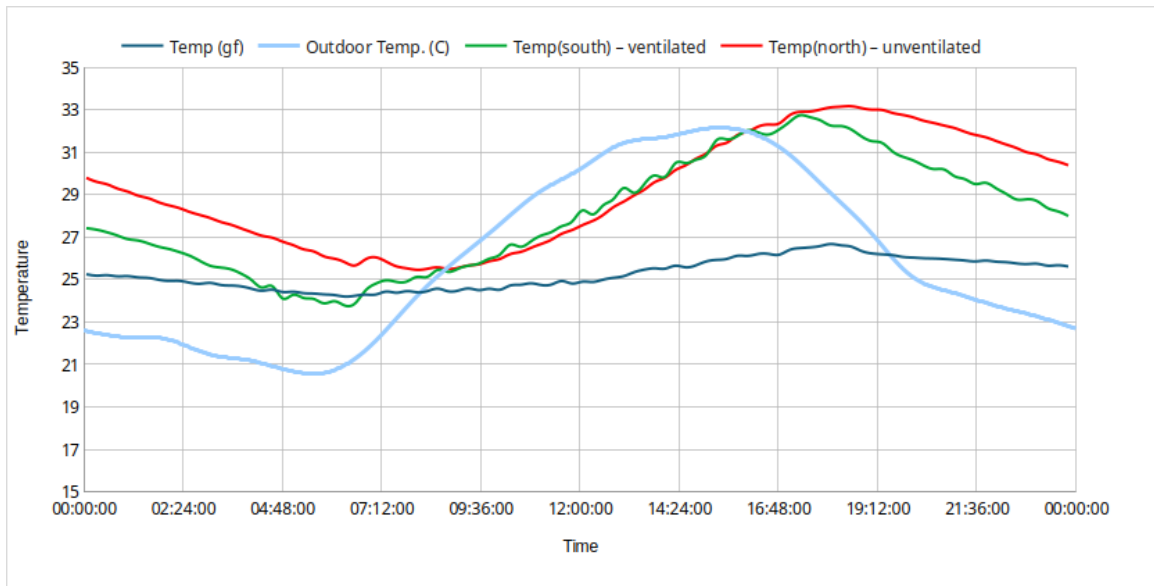


Figure 4: Temperatures ($^{\circ}\text{C}$) over 24 hours during summer (30/11/2019) with the roof painted gray and the southern vault ventilated and the northern unventilated. The north side is consistently hotter at night, but as the outside temperature rises both vaults are about the same temperature.

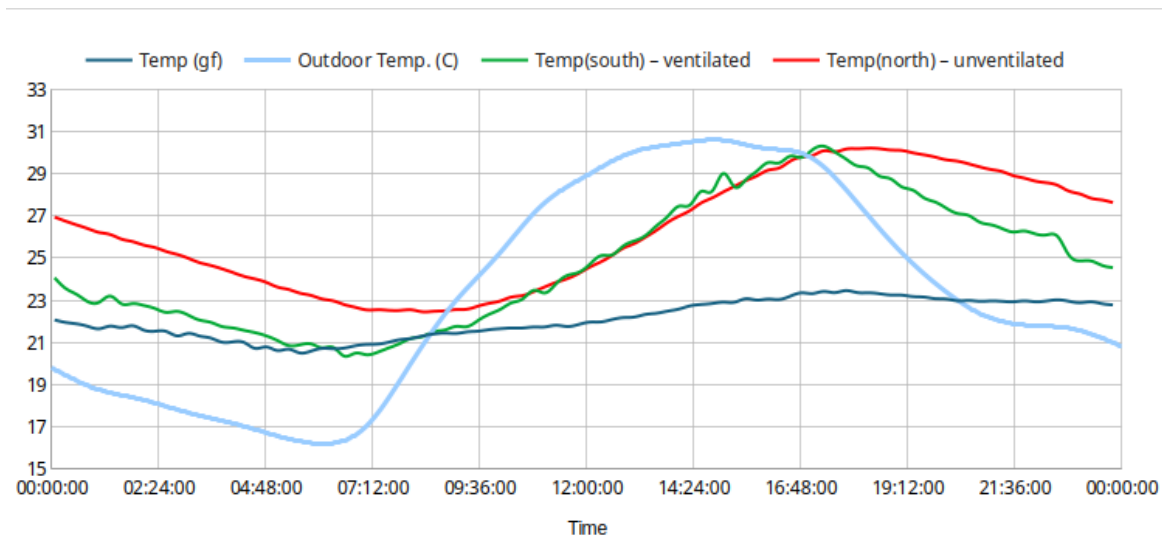


Figure 5: Temperatures ($^{\circ}\text{C}$) over 24 hours during spring (28/10/18) with both vaults painted maroon. The south side is ventilated and the north is not. The north remains consistently hotter except during the heating phase during the day when the two vaults are about the same temperature.

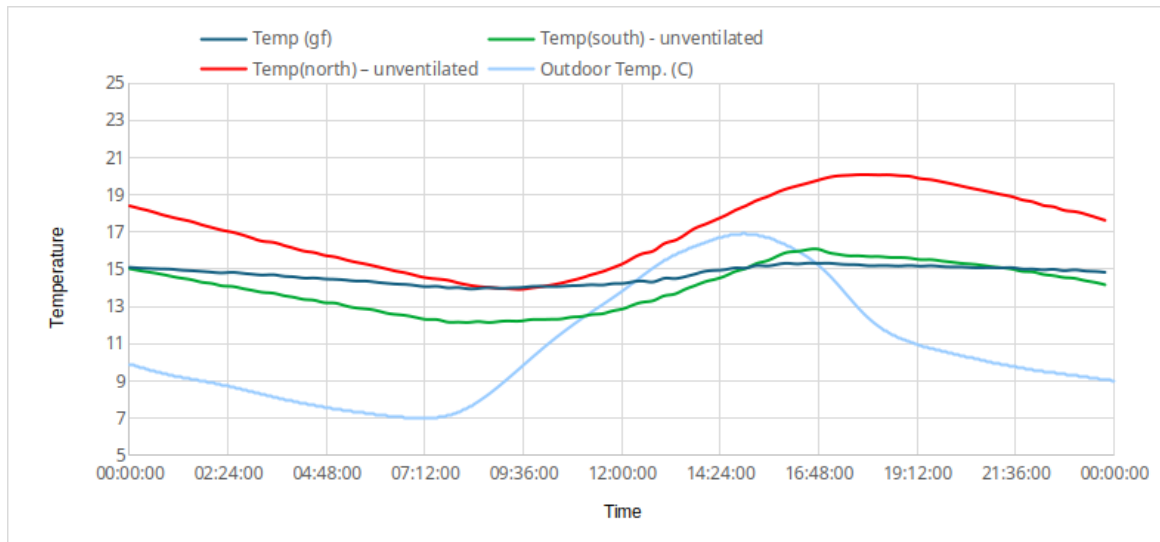


Figure 6: Temperatures ($^{\circ}\text{C}$) over 24 hours during winter (21/06/18) with both vaults painted maroon and both vents open. The north side is consistently 4°C warmer for the whole 24 hours.

Table 1: Observations of the effect of the roofing from data provided.

Grey roof	Late in the year, Oct-Nov, 2019 Warm weather South ventilated North unventilated	At night hotter inside than out. Ground floor stable around 25°C Lag around 2 hours during heating, but much longer in cooling phase South side cooler at night, but same during the day Maximum temperatures inside match outside (with lag) With no ventilation, north and south temperatures are closely aligned.
White roof	Summer - hot day South ventilated North unventilated	Ground floor very stable around 25°C South hotter during the day if ventilated South cools more at night if ventilated Lag during heating phase is about 4.5 hours
	Winter - cool day Both unventilated	Ground floor stable around 14°C South side consistently 2°C cooler all day Maximum. temperature inside matches outside (with lag) At night, insider 4°C warmer than outside.
Maroon roof	Winter - cool day	Cooler inside than out during the day, warmer at night Lag of about 3.5 hours during heating Ground floor stable around 14°C Unventilated $\Rightarrow$ South side cooler at night About the same during the day. Indoor about 5°C warmer than outside at night North side consistently warmer during the day

Some more detailed observations are given in Table 1. There is still a lot of data to be studied and analyzed. Ideally, data including a consistent set for elimination should be considered. For example, days with similar temperatures with all windows open, all closed, north open with south closed and vice versa. Other available meteorological data such as wind speed would also be useful on days where windows are open, since this would allow the impact of flow of air through the open windows (for example, adding heat during the day and extra cooling at night).

3 The one-dimensional model

A complete model of the structure's thermal properties would require a full simulation using finite elements or some other high level numerical solution method. To set up these equations we follow the model of Myers et al. [1]. An initial attempt at this will be described in the next section. Here, we devise a simpler model that reproduces the behaviour of the conditions in the house throughout the year without a lot of detail. The approach is to assume a one dimensional house where the different regions are modelled using simplified procedures and assumptions about the rate of heat transport. A 'shape factor', designed to represent different shaped roofs, is incorporated. As a one dimensional model it is faster to do the simulations and easier to understand the outputs, clarifying the different effects on the thermal behaviour.

3.1 One-dimensional equations

Assuming the house is a single vertical structure consisting of different properties, representing the walls, the first-floor room, and the roof, we can write the equations for the temperature, $u(x, t)$, within the roof as;

$$\rho C_p \frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial z^2}, \quad 0 < z < L, t > 0, \quad (1)$$

with boundary conditions

$$\begin{aligned} -k \frac{\partial u}{\partial z} \Big|_{z=L} &= q_{conv} + q_{sun} + q_{rad} \\ &= -\alpha_R (u(L, t) - u_a) \\ &+ \gamma(1 - A) Q_{max} \sin \left(\frac{2\pi t}{p_D} \right) - \varepsilon \sigma (u(L, t)^4 - u_a^4) \end{aligned} \quad (2)$$

$$-k \frac{\partial u}{\partial z} \Big|_{z=0} = \alpha_R (u(0, t) - u_{room}) \quad (3)$$

and for a room situated beneath, the temperature $u_{room}(t)$ can be described by

$$-k \frac{du_{room}}{dt} \Big|_{z=0} = -\frac{\alpha_R}{V} (u(0, t) - u_{room}) - \frac{\alpha_W}{V} (u_{room}(t) - u_a(t)) \quad (4)$$

where quantities are defined in Table 2. The first term on the right side of (4) is transfer of heat from the bottom of the roof into the upper rooms (inside the roof cavity) and the last term is to model transfer of heat between the room and outside (for example due to open windows).

Table 2: Table of variables and parameters

α_R	transfer coefficient between roof and the air (W/m^2)
ε	emissivity
β	diffusivity of the roofing material
A	albedo of the roof
σ	Stefan-Boltzman constant
γ	shape factor for the roof
L	thickness of the roof (m)
p_D	number of hours of daylight
ρ	density of the roof (kg/m^3)
C_p	the specific heat of the roof at constant pressure ($J/kg\ ^\circ C$)
k	thermal conductivity ($W/m\ ^\circ C$)
Q_{max}	irradiance from the sun (W/m^2).
q_{sun}	heat flux from the sunlight (W/m^2)
q_{conv}	heat flux from convection (W/m^2)
q_{rad}	radiative heat transfer (W/m^2)
$u(z, t)$	temperature in the roof ($^\circ C$)
$u_a(t)$	air temperature outside the house ($^\circ C$)
$u_{room}(t)$	air temperature inside the house ($^\circ C$)
u_0	typical value of temperature, ($0^\circ C$)
u_{ave}	approximate average temperature ($20^\circ C$)
V	volume of the room under the roof (m^3)
z	vertical coordinate (m)
t	time (s)
τ	typical time scale (3600s)

Equation (1) is the diffusion equation, computing the redistribution of heat due to the conditions above and below the roof. The influx of heat from above the roof (2) consists of the impact of sunlight (q_{sun}), loss due to convection (q_{conv}), and radiative transfer (q_{rad}). At the bottom of the roof heat is exchanged convectively with the upper room (equation (3)) with u_a being the ambient (outside) temperature. The influx from sunlight is dependent on the number of daylight hours, and is zero during the night. Heat exchange between the room and the underside of the roof is modelled by (equation (4)). The heat entering needs to be spread throughout the room (assuming sufficient mixing in the air). The extra term in (equation (4)) is due to having open or closed windows. If the windows are closed then the room is assumed to be insulated so the only method of heat exchange is through the roof. A further equation could be added to include heat transfer with the ground floor, but this is not included in this work.

The number of parameters in the problem is reduced if we nondimensionalize with respect to length, L , time, τ , and scale temperature by letting

$$\hat{u} = \frac{(u - u_0)}{u_{ave} - u_0}.$$

Assuming the maximum variation in temperature is about $20^\circ C$ we can take $u_{ave} - u_0$ to be 20.

If we take u_0 to be $0 \text{ deg } C = 273^\circ K$, then $\hat{u} = 1$ corresponds to $20^\circ C$ and $u = 2$ corresponds to $\hat{u} = 40^\circ C$. In non-dimensional variables the problem then becomes;

$$\frac{\partial \hat{u}}{\partial \hat{t}} = \mu_1 \frac{\partial^2 \hat{u}}{\partial \hat{z}^2}, \quad 0 < \hat{z} < 1, \hat{t} > 0, \quad (5)$$

with boundary conditions

$$\frac{\partial \hat{u}}{\partial \hat{z}} \Big|_{\hat{z}=1} = \mu_2 (\hat{u}(1, t) - \hat{u}_a(t)) + \gamma(1 - A)\mu_3 \sin\left(\frac{2\pi \hat{t}}{T}\right) + \mu_4 (\hat{u}(1, \hat{t})^4 - \hat{u}_a(t)^4), \quad (6)$$

$$\frac{\partial \hat{u}}{\partial \hat{z}} \Big|_{\hat{z}=0} = -\mu_2 (\hat{u}(0, t) - \hat{u}_{\text{room}}(t)) \quad (7)$$

and

$$\frac{d\hat{u}_{\text{room}}(t)}{d\hat{t}} \Big|_{\hat{z}=0} = \mu_5 (\hat{u}(0, \hat{t}) - \hat{u}_{\text{room}}(t)) - \mu_6 (\hat{u}_{\text{room}}(t) - \hat{u}_a(t)) \quad (8)$$

where

$$\begin{aligned} \mu_1 &= \frac{\tau \beta}{L^2 \rho C_p}, \quad \mu_2 = \frac{\alpha_R L}{k}, \quad \mu_3 = \frac{Q_{\max} L}{k}, \\ \mu_4 &= \frac{\varepsilon \sigma u_0^3 L}{k}, \quad T = p_D / \tau, \quad \mu_5 = \frac{\alpha_{Ra} \tau}{kV}, \quad \mu_6 = \frac{\alpha_W \tau}{kV}. \end{aligned}$$

Approximate values of some of the parameters are; $\varepsilon \approx 0.5$, $\sigma \approx 6 \times 10^{-8}$, $L \approx 0.17$, $k \approx 1$, $\frac{\beta}{\rho C_p} = D_R \approx 1$ and we are going to choose $\tau = 3600$, that is, 1 hour, meaning that we can estimate $\mu_1 \approx 0.15$, $\mu_2 \approx \mu_5 \approx 1.5$ and $\mu_4 \approx 10^{-5}$.

3.2 Shape Factor

The quantity γ comes from the sun's angle above and the shape of the roof of the hut. Suppose that the roof has shape $z = f(y)$, then the normal at any point on the roof is

$$\nabla(z - f(y)) = \mathbf{k} - f'(y)\mathbf{j}.$$

If the sun is directly above, then the component of sunlight normal to the surface can be obtained as the dot product of the unit normal $\hat{\mathbf{n}}$ with $\mathbf{k}$ integrated over the surface, that is,

$$\int_S \frac{1}{\sqrt{1 + f'(y)^2}} ds = \int_0^{2R} dy,$$

noting that

$$ds = \sqrt{1 + f'(y)^2} dy,$$

so that the sunlight is equivalent to that landing on a flat slab of length $2R$, where R is the half-width of the roof space. However, we note that this heat must be distributed over the surface area of the roof itself. Therefore, a shape factor γ can be written as

$$\gamma = \frac{2Rl}{\text{Surface area of the roof}}.$$

As an example, for a circular arched roof, where l is the length of the house, and the sun is passing lengthwise along the house, then

$$\gamma = \frac{2Rl}{\pi Rl} = \frac{2}{\pi}$$

while for a quadratic arch, the quantity would be

$$\gamma = \frac{2Rl}{2l \int_0^R (R^2 - y^2) dy} = \frac{2Rl}{\frac{4}{3}R^3l} = \frac{3}{2R^2}.$$

In the case of a flat roof, $\gamma = 1$. The total energy absorbed would be the same but it is distributed over a larger area. This effect would be exacerbated if the sun is lower in the sky and the θ .

3.3 Results of the one-dimensional model

A series of simulations was performed using this one-dimensional model to test if we could replicate the trends shown in the data and then try different scenarios to test the heating or cooling outcomes. The equations were solved numerically using a forward-time, centred-space (FTCS) method. This is not the most accurate method but it is fast to set up and run, so quickly provides some idea of the outputs. Since this model is only able to consider a single side of the house, we did simulations for summer and winter with a white and grey roof, considering a flat or domed structure. No ventilation examples were considered, meaning that $\mu_6 \approx 0$. It would be possible to couple this version with a second one-dimensional part of the house with interaction terms between the two, but this is a possible subject for further work.

In the summertime simulation, daylight time was set to 12 hours, while in winter it was set to 8 hours. The coefficient of sunlight was set to $mu_3 = 3.0$ for summer and $\mu_3 = 2.0$ for winter. Albedo was set for a white (0.75) or grey (0.3) roof and the shape parameter was set for a circular dome or a flat roof.

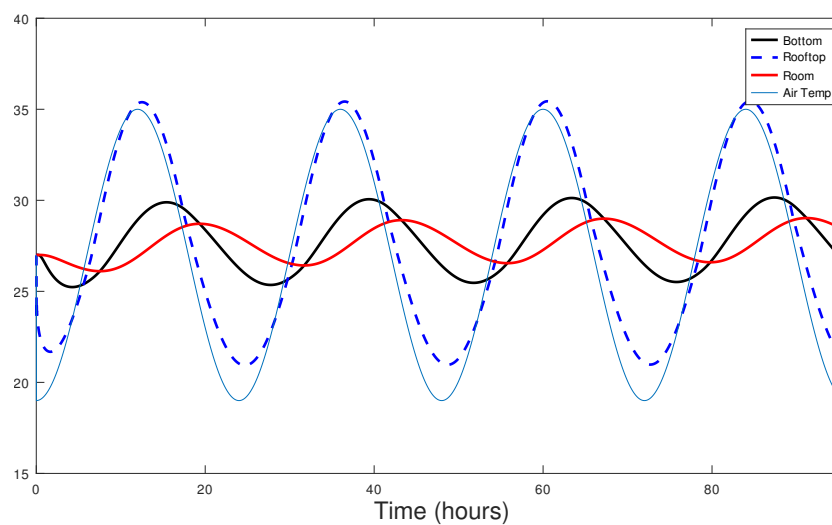
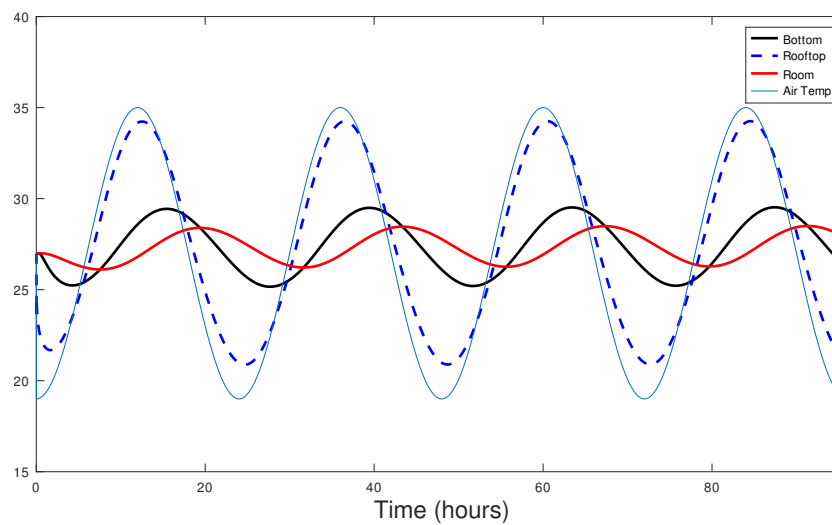


Figure 7: Summer with a domed roof - Top: white, bottom: grey. The only difference is the roof colour, modelled as albedo of 0.75 and 0.3 respectively.

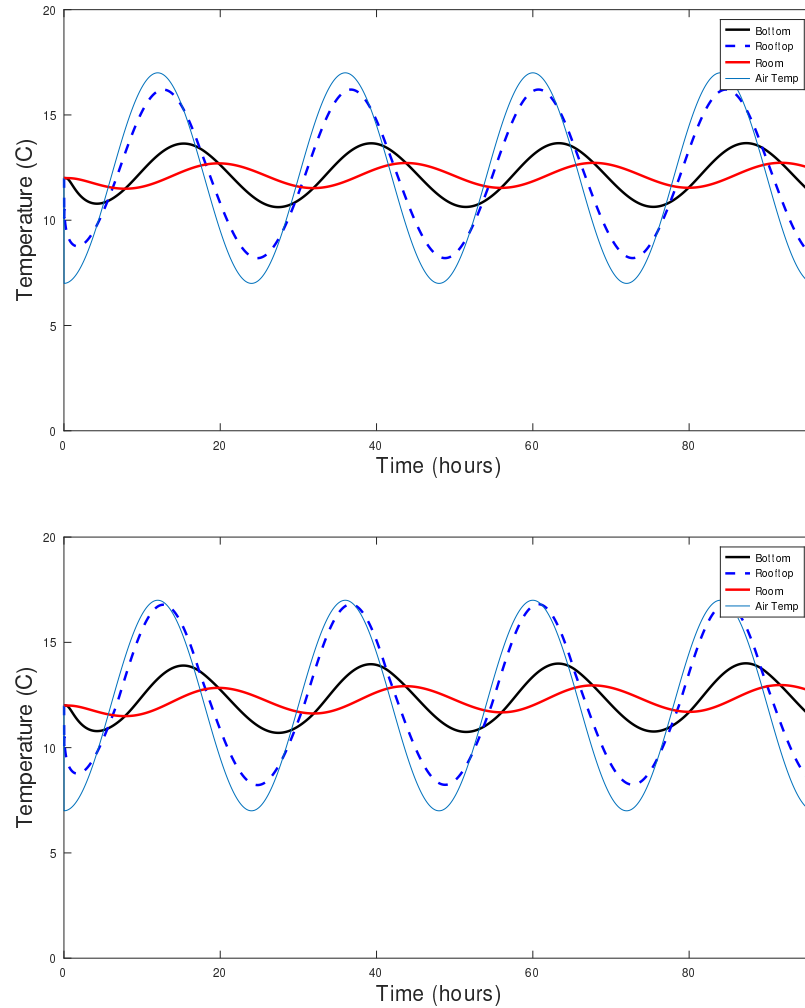


Figure 8: Winter simulation with circular dome - Top: white roof, bottom: grey roof

Figure 7 shows the summer case with a white, circular domed roof (top) and a similar roof painted grey (bottom), over a period of 4 days. The only difference between these two simulations is the colour of the roof, modelled by the albedo, set to be 0.75 for the white roof and 0.3 for the grey. The first couple of days vary slightly until the model settles into a repeated pattern. The curves show the temperature in degrees C outside, atop the roof, on the underside of the roof and in a room beneath. The temperature in the room beneath would be something like the temperature in the upstairs part of the house. Downstairs is not considered here. The temperature outside and on the top of the roof follow quite closely with a delay, while the temperature underneath the roof is delayed about 2 hours from the outside roof temperature and is also damped in amplitude as the heat diffuses through. The temperature in the room follows a similar pattern but is damped slightly by the insulating effect of the roof. In the case of the white roof the temperatures in the house are about 1 degree lower than for the grey. It is also interesting that the temperature atop the roof reaches above the outside temperature when the roof is grey due to the extra absorption of heat. The minimum temperatures are about half a degree lower in the case of the white roof.

Summer	$\mu_3 = 3.0$	Day length = 12 hours	Outside Temp	19–35
White, Albedo=0.75	Flat Min	Flat Max	Dome Min	Dome Max
Rooftop	20.98	34.7	20.85	34.3
Under roof	25.2	30	25.1	29.7
Room	25.8	29.1	26.5	28.6
Grey, Albedo=0.3,	Flat Min	Flat Max	Dome Min	Dome Max
Rooftop	21.1	36.8	20.9	35.7
Under roof	25.7	31.2	25.5	30.25
Room	26.7	30.3	26.7	29.2
Winter	$\mu_3 = 2.0$	Daylength=8 hours	Outside temp	7-17
White, Albedo=0.75	Flat Min	Flat Max	Dome Min	Dome Max
Rooftop	8.7	16.3	8.2	16.2
Under roof	10.8	13.8	10.6	13.7
Room	11.4	13	11.6	12.8
Grey, Albedo=0.3	Flat Min	Flat Max	Dome Min	Dome Max
Rooftop	8.3	17.3	8.2	16.8
Under Roof	10.7	14.4	10.7	14
Room	11.6	13.5	11.7	13

Table 3: Results of simulations for temperature ranges on the roof, under the roof and in a room beneath. Cases are for winter and summer, with a flat or domed roof.

These results are qualitatively consistent with the measurements, although further tuning of the model and use of real ambient temperature data would be necessary to provide a better match.

Figure 8 shows the same house conditions but for a winter scenario. In this case, the white roof generates a colder internal environment, again due to more heat being reflected. In this case, the white roof again has maximum temperatures lower than the grey roof, but this is now less desirable because of the colder conditions in winter. The minimum temperatures show the same trend as the results for summer, with the white roof leading to slightly lower indoor temperatures.

The results are typical of other scenarios considered, and the maxima and minima for the different cases are shown in Table 3.

In the time available in the study group, it was not possible to include all important effects, nor to tune the various parameters to reflect the real situation, but the model results do provide some insight into the impact of the different features of the housing. To compare with the informal housing, a reasonable approximation might be that the temperature in such structures would be close to the ambient temperatures. Therefore, in summer, the white domed roof provides an upstairs temperature 4-5 degrees cooler, while in winter the minimum is 5-6 degrees higher. A more accurately tuned model could be used to analyse the impact of different features much more closely, potentially leading to a better design.

4 Direct simulations

In this final section, results using a finite element model are described and set up to simulate a full two-dimensional double-domed house. The partial differential equation solver in **Mathematica** was set up to simulate the temperature in the double-vaulted house. Equation (1) was solved subject to Dirichlet conditions on the outside of the building,

$$u(x, z, t) = \theta u_a \text{ on } z = f(x), \quad (9)$$

where $f(x)$ is the shape of the roof and θ is an empirical factor to account for heat transfer to the outside of the roof. No flux of heat is allowed through the base of the structure,

$$\frac{\partial u}{\partial z} = 0 \text{ on } z = 0, \quad (10)$$

an assumption that is reasonable since little heat would be lost (or gained) through the floor of the building.

The outside temperature readings, u_a , for a summer day and a winter day were provided. The measurements of the outside temperature were taken in hourly intervals and a Chebychev polynomial was used to construct a continuous curve to use in the numerical solver. This curve was used as the boundary condition for the roof. That is, we assumed that the roof's temperature was equal to that of the outside temperature. The initial temperature used was that of the initial temperature of the room and not that of the roof. The Implicit Differential-Algebraic solver was used in **Mathematica** by utilising **NDSolve**. The PDE (1) in two space dimensions, was discretised and solved using the method of lines, where a finite element method was used to discretise the spatial coordinates producing the meshed region as shown in Figure 9. The thermal conductivity value used was halfway between that of air and cement. Note that no convection of the air is included in this simulation, and this is therefore a good option for future work, since movement of the air in the room will undoubtedly redistribute the heat.

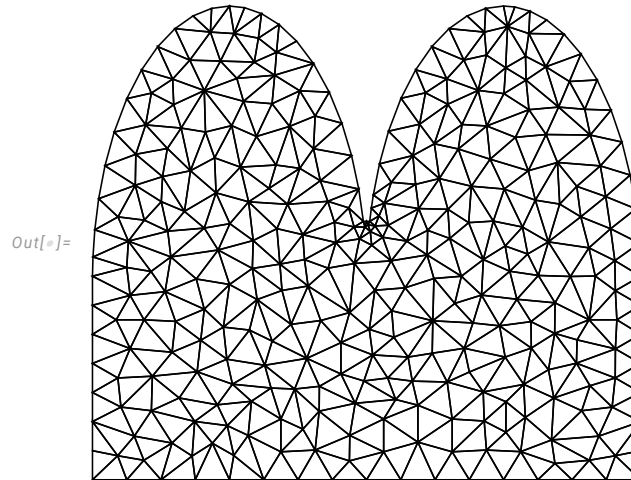


Figure 9: Finite element mesh of double roof building

Depending on the model of thermometer used, the error associated with the temperature readings can be as great as $\pm 2^\circ\text{C}$. We also know that the temperature of the roof may differ

greatly from that of the outside temperature. Taking this into account, we can adjust the boundary condition of the roof to hopefully improve results. The temperature on the roof was adjusted to provide a more accurate internal temperature, that is, θ was adjusted to provide a better match to the internal temperatures.

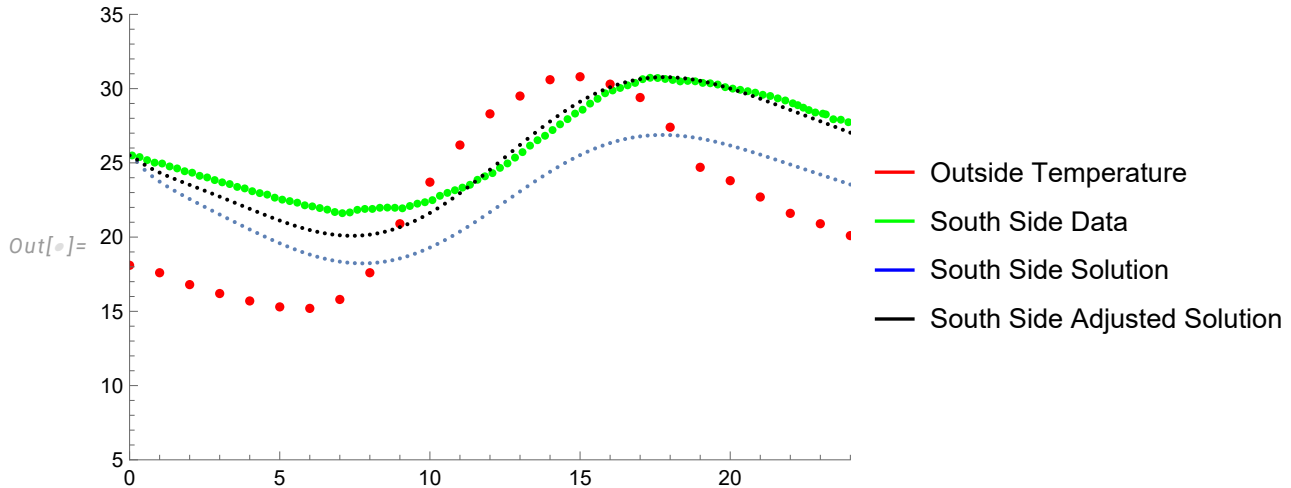


Figure 10: Numerical summer solution with adjusted roof temperature.

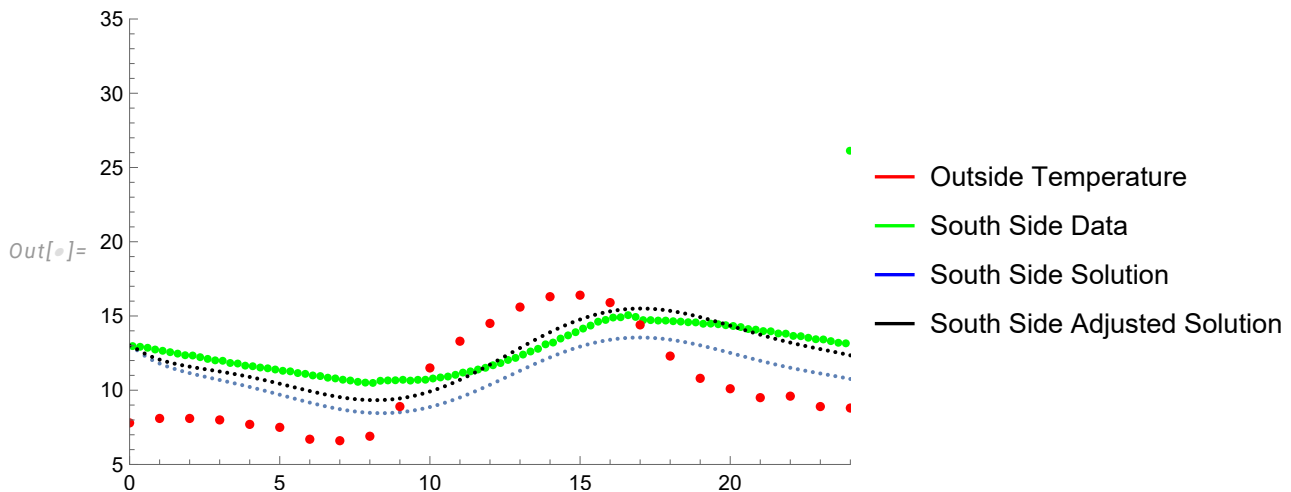


Figure 11: Numerical winter solution with adjusted roof temperature.

Figures 10 and 11 show that adjusting the boundary condition resulted in a solution that better reflects the internal temperature reading within the room. The model of the house sufficiently captures the behaviour of the temperature in the evening but underestimates the early morning temperatures. The model also performs better in winter, which is probably due to lower roof temperatures carrying over from the previous day. This informs us that the following improvements can be made:

1. The roof, walls, and rooms need to be modelled as separate regions with their own properties.

2. The early morning to midday temperatures are under predicted because the radiation of the residual heat stored in the roof overnight needs to be included.
3. It might be necessary to include ventilation, but this would require a 3D model.
4. The roof needs to be divided into multiple sections that can account for shade depending on the position of the sun. This will also have an effect on the boundary conditions.
5. Air convection will redistribute the heat around the upper rooms, but this is not included in the simulation. Inclusion of convective transport of heat will improve the calculations.

An initial investigation of separating the room from the roof was also investigated. As a double dome is more complex, a single dome was used. Again, **Mathematica** was used to simulate the building and roof, converting geometric shapes into the meshed regions as seen in Figure 12 where the green area is the internal part of the room and the red area is the roof. According to the specifications of the building provided, a thickness of 17cm was used for the roof. Additionally, boundary region markers had to be defined as seen in Figure 13 where: 1 is the ground boundary, 2 is the inner boundary where the roof meets the room, 3 is the outer boundary where the roof meets the ambient air.

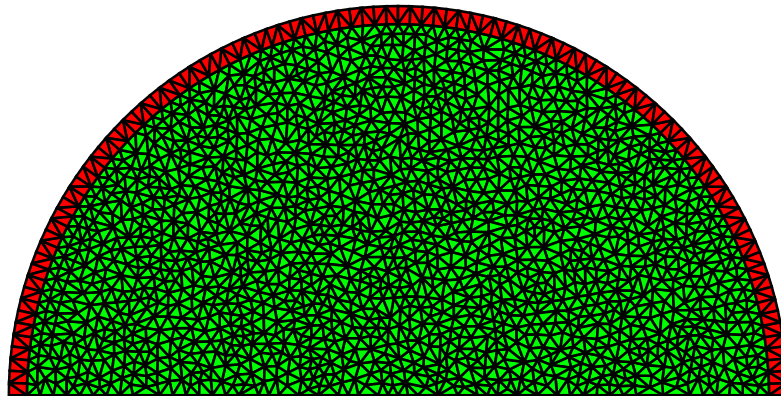


Figure 12: The finite element mesh of the roof.

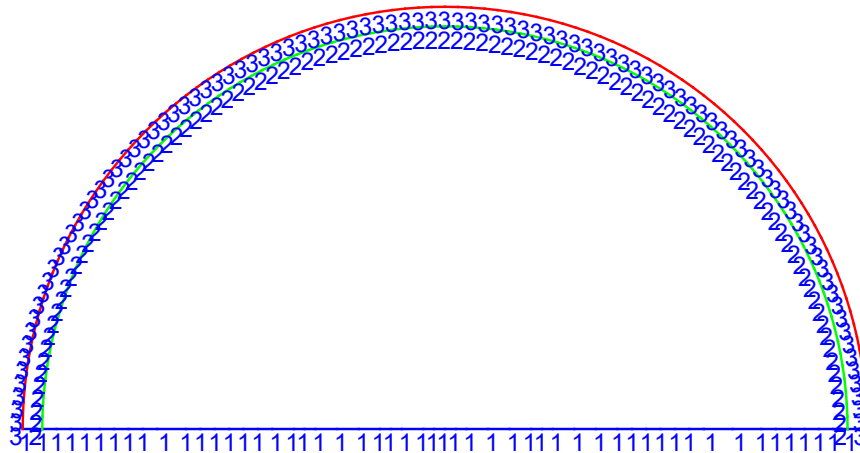


Figure 13: The boundary view of the roof, showing the regions with different conditions.

Again, `NDSolve` was used, but two separate thermal conductivities were used. In the green meshed region a thermal conductivity equal to that of air was used, and in the red meshed region the thermal conductivity of cement was used. Cement was chosen because the structure is constructed from repurposed material that includes brick and concrete. Two simulations were performed, with and without an adjustment for outside temperature readings. Only a simulation for summer was conducted.

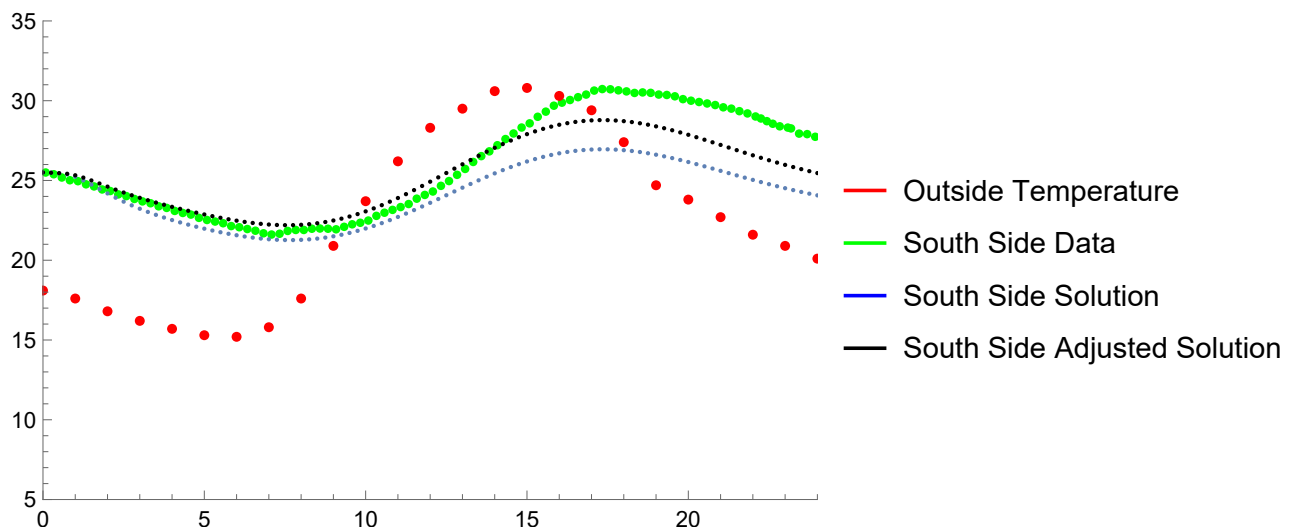


Figure 14: Numerical summer solution with roof modelled separately.

As seen in Figure 14, modelling the roof to be separate from the room better predicted the daytime temperature readings. However, the model now predicts evening temperatures that are too low.

This full two dimensional model of the domed structure using finite elements has shown the possibility of providing a complete simulation of the situation in the low-cost housing prototype, but it is in the early stages and would require more development to include appropriate boundary conditions of heat exchange and sunlight.

5 Final comments

The two models both produced promising results when compared to the available data. It would be expected that both could be improved by calibration and fine tuning of the model parameters. In the case of the one-dimensional model, incorporation of a second one-dimensional column coupled to the first, could be used to represent the other side of the house, while the inclusion of the heat sink of the roof and appropriate boundary conditions on the upper part of the roof in the finite element model is likely to provide much more accurate comparisons and a more useful design tool. Therefore, a merging of these two approaches should produce a reliable tool to model the thermal properties of the house throughout the year and a useful tool for optimizing the conditions inside.

The project was successful in developing approaches that could provide useful information for the industry partners in understanding the impact of the competing effects on the thermal behaviour of the structure, and hence in future design improvements.

References

- [1] Agualeles M, Calvo-Schwarzwalder M, Font F, Myers TG, 2023, A mathematical model for the energy stored in green roofs, *Applied Math. Modelling*, 115, 513-540.